# How does <br> FORRESTER HIGH SCHOOL <br> Do Numeracy? 



NUMERACY HANDBOOK
A guide for students, parents and staff

## Introduction

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## What is the purpose of the booklet?

This booklet has been produced to give guidance and help to staff, students and parents. It shows how certain common Numeracy topics are taught in mathematics and throughout the school. It is hoped that using a consistent approach across all subjects will make it easier for students to progress.

## How can it be used?

The booklet includes the Numeracy skills useful in subjects other than mathematics.

It is intended that staff from all departments will support the development of Numeracy by reinforcing the methods contained in this booklet. If this is not possible because of the requirements of your subject, please highlight this to students and inform a member of the Numeracy group, so that the booklet can be updated to include this information next session.

```
NOTE: }\underline{3}\mathrm{ means 3 parts out of a total of 4
also }\quad\frac{3}{4}\mathrm{ means 3 }\div4=0.7
    4
```

It should be noted that the context of the question, whether a calculator is permitted or not and the nature of the numbers involved has the potential to change the given level.

## Why do some topics include more than one method?

In some cases (e.g. percentages), the method used will be dependent on the level of difficulty of the question, and whether or not a calculator is permitted.

For mental calculations, pupils should be encouraged to develop a variety of strategies so that they can select the most appropriate method in any given situation.
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Addition

## Mental strategies



There are a number of useful mental strategies for addition. Some examples are given below.

Example 1 Calculate $54+27$
Eevel 2
Method 1 Add tens, then add units, then add together
$50+20=70$
$4+7=11$
$70+11=81$

Method 2 Split up number to be added into tens and units and add separately.
$54+20=74 \quad 74+7=81$

Method 3 Round up to nearest 10 , then subtract
$54+30=84$ but 30 is 3 too much so subtract 3 ;
$84-3=81$

## Addition

## Written Method



When adding numbers, ensure that the numbers are lined up according to place value. Start at right hand side, write down units, carry tens

Example 2 Add 3032 and 589
Level 2


When adding decimals we make sure all the decimal points are lined up.

Example 3 Add $43.8+4+23.76$
Level 2


Remember you can add as many numbers together in a single sum as you like.

Subtraction

## Mental Strategies



There are a number of useful mental strategies for subtraction. Some examples are given below.

Example 1 Calculate 93-56
Level 2
Method 1 Count on

Count on from 56 until you reach 93 . This can be done in several ways e.g.


Method 2 Break up the number being subtracted

$$
\text { e.g. subtract } 50 \text {, then subtract } 6 \quad 93-50=43
$$

$$
43-6=37
$$



## Written Method (We do NOT "borrow and pay back")

 subtraction (see below). Alternative methods may be used for mental calculations.

Example 2 4590-386
Level 2
$45^{8} 910$

- 386

4204

Example 3 Subtract 692 from 14597
$14{ }^{3} 597$

- 692

13905

Example 4 Find the difference between 327 and 5000 Level 2


We need to "BUMP" our borrow 1 back to the end.


When subtracting decimals we make sure all the decimal points are lined up.

Example 5 Subtract 8.36 from 20.9
Level 2

$$
\begin{array}{r}
80.80 \\
-\quad 8.36 \\
\hline 12.54 \\
\hline
\end{array}
$$



Remember you can only have TWO numbers in a single subtraction calculation.

## Mental Strategies



It is essential that all of the multiplication tables from 1 to 10 are known. These are shown in the tables square below.

| $\times$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 |
| 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 |
| 10 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

Example 1 Find $39 \times 6$

## Method 1

Method 2



Multiplication
Multiplying by multiples of 10 and 100


To multiply by 10 you move every digit one place to the left.
To multiply by 100 you move every digit two places to the left.

Example 2 (a) Multiply 354 by 10
(b) Multiply 50.6 by 100

Level 2

$354 \times 10=3540$
$50.6 \times 100=5060$

Level 3
(c) $35 \times 30$
(d) $436 \times 600$

| To multiply by 30 , multiply |
| :--- |
| by 3 , then by 10 . |

To multiply by 600, multiply by 6 , then by 100 .
$35 \times 3=105$
$436 \times 6=2616$
$105 \times 10=1050$
$2616 \times 100=261600$
So $35 \times 30=1050$
So $436 \times 600=261600$

Example 3
(a) $30 \times 60$

Level 3
(b) $20 \times 700$
$2 \times 10 \times 7 \times 100$
$=14 \times 10 \times 100$
$=18 \times 10 \times 10$
$=14000$

We may also use these rules for multiplying decimal numbers.

Example 4
(a) $2.36 \times 20$
(b) $38.4 \times 50$

Level 3
$2.36 \times 2=4.72$
$38.4 \times 5=192.0$
$4.72 \times 10=47.2$
$192.0 \times 10=1920$
So $2.36 \times 20=47.2$
So $38.4 \times 50=1920$

## Written Method

Example 5 Multiply 246 by 8
Level 2


Example 6 Multiply 4367 by 50
Level 3
4367

$\times$| 43350 |
| :--- |
| 218350 |


| $\times 50$ is the same as $\times 5 \times 10$ |
| :--- |
| Put the 0 into the answer first $(\times 10)$ |
| then multiply by 5 |

Example 7 Multiply 472 by 300
Level 3


## Long Multiplication



We can multiply by a 2 or 3 digit number by combining the above methods.

Example 8 Multiply 5246 by 52
Level 3


$$
\begin{array}{r}
5246 \\
\times \quad 52 \\
\times 10492 \\
\hline 26^{1} 2^{2} 3^{3} 00 \\
\hline 272492 \\
\times 50 \\
\times 52 \quad \begin{array}{l}
\text { To get the final answer add the two previous answers } \\
\text { together. }(50+2=52)
\end{array} \\
\hline
\end{array}
$$

Alternatively we could set it out as follows:



To multiply decimals we ignore the decimal points) until after we multiply. The points) are not necessarily lined up when setting the question out.
The decimal point gets placed in the answer after multiplication is complete.

## Example $923.76 \times 6$

Level 2


Example $10134.9 \times 0.3$ Level 3


Example $11132.8 \times 3.4$


Level 3


Dividing by multiples of 10 and 100


To divide by 10 you move every digit one place to the right.
To divide by 100 you move every digit two places to the right.

Example 1 (a) $8450 \div 10$
(b) $37.9 \div 100$

Level 2

$8450 \div 10=845$
(c) $440 \div 40$

To divide by 40 , divide by 4 , then by 10 .

$$
\begin{aligned}
& 440 \div 4=110 \\
& 110 \div 10=11 \\
& \text { So } 440 \div 40=11
\end{aligned}
$$


$37.9 \div 100=0.379$
(d) $85.6 \div 200$

To divide by 200, divide by 2 , then by 100 .
$85.6 \div 2=42.8$
$42.8 \div 100=0.428$
So $85.6 \div 200=0.428$

Division
You should be able to divide by a single digit or by a multiple of 10 or 100 without a calculator.

## Written Method

Example 2 There are 192 pupils in first year, shared equally between 8 classes.
Level 2 How many pupils are in each class?
24
$8 \longdiv { 1 9 ^ { 3 } 2 } \quad$ There are 24 pupils in each class

Example 3 Divide 4.74 by 3
Level 2
$3 \longdiv { 4 . 5 8 }$

When dividing a decimal by a whole number, the decimal points must stay in line.

Example 4 A jug contains 2.2 litres of juice. If it is poured evenly into 8 glasses, how much juice is in each glass?

8 \begin{tabular}{|l|l|}
\hline 0.275 <br>

$2.2^{2} 2^{6} 0^{4} 0$ \& | If you have a remainder at the end of a |
| :--- |
| calculation, add a zero onto the end of the decimal |
| and continue with the calculation. 2.20 is the same |
| as 22.2 Continue to add 0 's as reauired | <br>

\hline
\end{tabular}

Each glass contains 0.275 litres

Example 5 Divide 575 by 4
Level 3 $4 \longdiv { 1 4 3 . 7 5 }$

If there is no decimal point then put the point in place before you add the first zero. 575.0 is the same as 575 .
Continue to add O's as required.

Division


Example $6467400 \div 40$
Level 3

We don't want to divide by 40 . We can turn 40 into 4 by dividing it by 10 .
If we do this, to balance the calculation, we must also divide 467400 by 10
$4 \longdiv { 4 6 ^ { 2 } 7 ^ { 3 } 4 ^ { 2 } 0 }$
The answer to both calculations will be the same if the calculation has been adjusted AND balanced.

So $467400 \div 40=11685$

Example 7 Divide $238.2 \div 300$
Level 3


So $238.2 \div 300=0.794$

Example 8 Divide 357.9 by 0.6
Level 3
 $6 \longdiv { 3 ^ { 3 } 5 ^ { 5 } 7 ^ { 3 } 9 . . ^ { 3 } 0 }$

So $357.9 \div 0.6=596.5$

## Long Division



We CAN divide by a 2 or 3 digit number without a calculator.

Example $93741 \div 32$


Level 3
Method 1


Possibly an easier way may be to treat the long division as a normal short division calculation and list the tables at the side.

Method 2

$\begin{array}{lll}32 & 32 & 32\end{array}$
6464
$96 \quad 96$
128128
160160
192192 224
256
288
We only list the 32
times table as far as we
need to go. We can add
to it if we need to.
The list length will only
ever be a maximum of 9
numbers.

Example $10357.4 \div 4.6$
Level 3


When we have a never ending decimal as our answer we have to decide when to stop dividing and round appropriately (see rounding).

## Order of Operation (BODMAS)

Consider this: What is the answer to $2+5 \times 8$ ?
Is it $7 \times 8=56$ or $2+40=42$ ?
The correct answer is 42 .


Calculations which have more than one operation need to be done in a particular order. The order can be remembered by using the mnemonic BODMAS. The higher the level the higher the priority

The BODMAS rule tells us which operations should be done first.
BODMAS represents:

| (B)rackets | Top level |
| :--- | :--- |
| (O)f |  |
| (D)ivide Middle level <br> (M)ultiply  <br> (A)dd Bottom level <br> (S)ubract  |  |

Scientific calculators use this rule, some basic calculators may not, so take care in their use.

Example $1 \quad 15-12 \div 6$
Level $2=15-2$
= 13

Example $2 \quad(9+5) \times 6$
Level $4=14 \times 6$
$=84$

Example $318+6 \div(5-2)$
Level $4=18+6 \div 3$
$=18+2$
$=20$

Example $4 \quad 16+5^{2}$
Level $3=16+25$

$$
=41
$$

Example $5 \quad(4+2)^{2}$
Level $4=6^{2}$
$=36$

BODMAS says divide first, then subtract

Brackets first
then multiply.

Brackets first
then divide
now add
multiply first ( $5 \times 5$ )
then add
brackets first
then multiply $(6 \times 6)$

Step 1: write formula
Step 2: substitute numbers for letters
Step 3: start to evaluate (BODMAS)
Step 4: write answer

Example 1 Use the formula $P=2 L+2 B$ to evaluate $P$ when $L=12$ and $B=7$.
Level 3

$$
\begin{aligned}
& P=2 L+2 B \\
& P=2 \times 12+2 \times 7 \\
& P=24+14 \\
& P=38
\end{aligned}
$$

$\frac{\text { Example } 2}{\text { Level } 3}$ Use the formula $I=\frac{V}{R}$ to evaluate $I$ when $V=240$ and $R=40$ $I=\frac{V}{R}$
$I=\frac{240}{40}$
$I=6$

Example 3 Use the formula $F=32+1.8 C$ to evaluate $F$ when $C=20$ Level 3
$F=32+1.8 C$
$F=32+1.8 \times 20$
BODMAS rules come into play.
$F=32+36$
$F=68$


Example 1 Compare the following pairs of numbers.
Level 3
a) 3 and -4
b) -6 and 4
c) -8 and -3
$3>-4$
$-6<4$
$-8<-3$
> means less than

When we ADD a positive number we move RIGHT on our number line.
When we SUBTRACT a positive number we move LEFT on our number line.

## Example 2 Calculate:

## Level 3



START
Example 3 Calculate:
$\longleftarrow$ To subtract move left
Level 3


## Estimation : Rounding

Numbers can be rounded to give an approximation.

The number to the right of the place value to which we want to round tells us how to round.

Example 1
Level 2


2652 ) rounded to the nearest 10 is 2650 .

2 is to the right of the 10's column. Round down.

26552 rounded to the nearest 100 is 2700.55 is to the right of the 100's column. Round up.

2652 rounded to the nearest 1000 is 30006 is to the right of the 1000's column. Round up.

When rounding numbers that lie exactly in the middle it is convention to ALWAYS round UP.

Example 2345 to the nearest 10
Level 2
346 ) $=350$ to the nearest 10

In general, to round a number, we must first identify the place value to which we want to round.
We must then look at the next digit to the right (the "check digit").
If the "check digit" is less than $5(0,1,2,3,4)$ round down.
If the "check digit" is 5 or more $(5,6,7,8,9)$ round up.

The same principle applies when rounding decimal numbers.

Example 1 Round 1.5739 to 1 decimal places (1.d.p.)
Level 3
The $1^{\text {st }}$ number after the decimal point (5) is the position
of our place value. The rounded number lies between 1.5 and 1.6
The $2^{\text {nd }}$ number after the decimal point is a 7. (this is the "check digit"). 7 means round up.
$1.5739=1.6$ (1.d.p.)

Example 2 Round 6.4721 to 2 decimal places (2.d.p.)
Level 3
The $2^{\text {nd }}$ number after the decimal point (7) is the position of our place value. The rounded number lies between 6.47 and 6.48

The $3^{\text {rd }}$ number after the decimal point is a 2. (this is the "check digit"). 2 means round down.

$$
6.4721=6.47 \text { (2 d.p.) }
$$

Example 3 Round 19.49631 to 2 decimal places (2.d.p.)

$19.49631=19.50$ (2.d.p.) | The number lies between 19.49 and 19.50. |
| :--- |
| 6 to the right means we round up. |
| We must include the 0 at the end as we |
| require 2 numbers after the point. |

Some students need a bit more visual help. You could also use a mini number line.

| $0,1,2,3,4$ | lower |
| :--- | :--- |
| $5,6,7,8,9$ | upper |

## Example 1

1. $57739=1.6$ (1.d.p.)

Level 3

## 1.5

$1.6 \longleftarrow 7$ upper

Example 2
$6.47(2) 1=6.47$ (2 d.p.)
$6.47 \longleftarrow 2$ lower
Level 3
6.48

Using rounded numbers in calculations to check an answer allows us to judge whether our answer is sensible or not.

Example 1 Tickets for a concert were sold over 4 days. The number of tickets
Level 2 sold each day was recorded in the table below.
How many tickets were sold in total?

| Monday | Tuesday | Wednesday | Thursday |
| :---: | :---: | :---: | :---: |
| 486 | 205 | 197 | 321 |

Estimate: $\quad 500+200+200+300$ $=1200$

Calculate:
486
205
197
$\begin{array}{r}+321 \\ \hline\end{array}$
1209 Answer $=1209$ tickets (reasonable when compared to estimate).

Example 2 A bar of chocolate weighs 42g. There are 48 bars of chocolate in a box. What is the total weight of chocolate in the box?

Estimate $=50 \times 40=2000 g$
Calculate:

42

$$
\frac{x 48}{336}
$$

$$
\frac{1680}{2016}
$$

Answer $=2016 \mathrm{~g}$
(reasonable when compared to estimate).

Time


It is essential to know the number of months, weeks and days in a year, and the number of days in each month.

Time may be expressed in 12 or 24 hour notation.

## 12-hour clock

Time can be displayed on a clock face, or digital clock.


05: 15
These clocks both show fifteen minutes past five, or quarter past five.

When writing times in 12 hour clock, we need to add a.m. or p.m. after the time.
a.m. is used for times between midnight and 12 noon (morning)
p.m. is used for times between 12 noon and midnight (afternoon / evening).

$$
5.15 \mathrm{am} \text { or } 5.15 \mathrm{pm} \text { ? }
$$

## 24-hour clock



## Examples

Level 2

| 12 hr | 24 hr |
| :---: | :---: |
| 9.55 am | 0955 hours |
| 3.35 pm | 1535 hours |
| 12.20 am | 0020 hours |
| 2.16 am | 0216 hours |
| 8.45 pm | 2045 hours |

Time


It is important to be able to change between units of time. Hours to minutes and minutes to hours.

Students should recognise everyday equivalences.

Level 3

## MINUTES $\longrightarrow$ HOURS

| 15 mins | $\frac{15}{60} \mathrm{hr}$ | $\frac{1}{4} \mathrm{hr}$ | 0.25 hr |
| :--- | :--- | :--- | :--- |
| 30 mins | $\frac{30}{60} \mathrm{hr}$ | $\frac{1}{2} \mathrm{hr}$ | 0.5 hr |
| 45 mins | $\frac{45}{60} \mathrm{hr}$ | $\frac{3}{4} \mathrm{hr}$ | 0.75 hr |

Example 1 Change minutes into hours
Level 4


Example 2 Change hours into minutes Multiply by 60
Level 4
$0.6 \mathrm{hrs}=0.6 \times 60=36 \mathrm{mins}$
$0.35 \mathrm{hrs}=0.35 \times 60=21 \mathrm{mins}$

$=168 \mathrm{mins}$
$2 \mathrm{hrs}=2 \times 60=120 \mathrm{mins}$
$0.8 \mathrm{hrs}=0.8 \times 60=\frac{48 \mathrm{mins}}{168 \mathrm{mins}}$
or
$2.8 \mathrm{hrs}=2.8 \times 60=168 \mathrm{mins}$

## Distance, Speed and Time.



For any given journey, the distance travelled depends on the speed and the time taken. If we consider speed to be constant, then the following formulae apply:

$$
\begin{aligned}
\text { Distance } & =\text { Speed } \times \text { Time } \\
\text { Speed } & =\frac{\text { Distance }}{\text { Time }} \\
\text { Time } & =\frac{\text { Distance }}{\text { Speed }}
\end{aligned}
$$

## WHAT'S THE FORMULA?



Example 3 Calculate the speed of a train which travelled 450 km in 5 hours Level 3


Example 4 How long did it take for a car to travel 209 miles at an average speed Level 4 of 55 mph ?
$D=209$ miles $\quad S=55 \mathrm{mph}$

$T=\underline{D}$
S
$T=\frac{209}{55}$
$0.8 \mathrm{hrs}=0.8 \times 60$
$\mathrm{T}=3.8 \mathrm{hrs}$

$$
=48 \mathrm{mins}
$$

$\mathrm{T}=3 \mathrm{hrs} 48 \mathrm{mins}$

Fractions

## What is a Fraction?





If the numerator and the denominator are the same number we have 1 whole.

## Equivalent Fractions



Equivalent fractions are fractions that represent the SAME AMOUNT. To find an equivalent fraction we multiply or divide both the numerator and the denominator of a fraction by the SAME number.

Example 1 Find equivalent fractions

Level 2
(a)

(b)


## Simplifying Fractions



When we DIVIDE to find an equivalent fraction, it is called SIMPLIFYING. We can simplify (divide) repeatedly until the fraction is in its SIMPLEST FORM.

Example 2 Write $\frac{56}{72}$ in its simplest form
Level 3
72



Fractions

## Improper Fractions



A top heavy fraction is called an IMPROPER fraction and is greater than 1. A MIXED NUMBER has a whole number part and a fraction part.

Example 3 Change the improper fraction $\frac{32}{}$ to a mixed number.
Level 3


Example 4 Change the mixed number $3 \frac{5}{7}$ to an improper fraction Level 3


## A Fractions of a Quantity



Example 5 Find $\frac{1}{5}$ of $£ 150$
Level 2

$$
\begin{aligned}
& \frac{1}{5} \text { of } £ 150 \\
&= \frac{150}{5} \\
&= £ 30 \\
& \text { Find } \frac{3}{4} \text { of } 48
\end{aligned}
$$

$\frac{\text { Example } 6}{\text { Level 2 }}$ Find $\frac{3}{4}$ of 48


## Percentages: Non-Calculator

Percent means out of 100. The symbol for percent is: \%
A percentage can be converted to an equivalent fraction or decimal.

Level $2 \quad 36 \%$ means $\frac{36}{100}$

$$
36 \%=\frac{36}{100}=\frac{9}{25}=0.36
$$



## Common Percentages

Level 2
Some percentages are used very frequently. It is useful to know these as fractions and decimals.

| Percentage | Fraction |  | Simplest Form |
| :---: | :---: | :---: | :---: |
|  | $\frac{1}{100}$ | $\frac{1}{100}$ |  |
| $10 \%$ | $\frac{10}{100}$ | $\frac{1}{10}$ | 0.1 |
| $20 \%$ | $\frac{20}{100}$ | $\frac{1}{5}$ | 0.2 |
| $25 \%$ | $\frac{25}{100}$ | $\frac{1}{4}$ | 0.25 |
| $331 / 3 \%$ | $\frac{331 / 3}{100}$ | $\frac{1}{3}$ | $0.333 \ldots$ |
| $50 \%$ | $\frac{50}{100}$ | $\frac{1}{2}$ | 0.5 |
| $66 / 3 \%$ | $\frac{66^{2} / 3}{100}$ | $\frac{2}{3}$ | $0.666 \ldots$ |
| $75 \%$ | $\frac{75}{100}$ | $\frac{3}{4}$ | 0.75 |
| $100 \%$ | $\frac{100}{100}$ | 1 | 1 |

There are many ways to calculate percentages of a quantity. Some of the common ways are shown below.

## Non- Calculator Methods

## Method 1 Using Equivalent Fractions (use table on previous page)

Example 1 Find $25 \%$ of $£ 48$
Level 2

$$
\begin{aligned}
& 25 \% \text { of } £ 48 \\
= & \frac{1}{4} \text { of } 48
\end{aligned}
$$

$$
=\frac{48}{4}
$$

$$
=£ 12
$$

Method 2 Using 1\% (1\% - 9\%)
In this method, first find $1 \%$ of the quantity (by dividing by 100), then multiply to give the required value.

## Example 2 Find $9 \%$ of 200 g

Level 3

## Method 3 Using 10\% (10\% and multiples of 10\%)

This method is similar to the one above. First find $10 \%$ (by dividing by 10), then multiply to give the required value.

Example 3 Find $70 \%$ of $£ 35$
Level 3

$$
\begin{aligned}
& 10 \% \text { of } £ 35 \\
= & \frac{1}{10} \text { of } 35 \\
= & \frac{35}{10} \\
= & \text { so } 70 \% \text { of } £ 35=7 \times £ 3.50 \\
= & \\
= &
\end{aligned}
$$

$$
\begin{aligned}
& 1 \% \text { of } 200 g \\
& =\frac{1}{100} \text { of } 200 \\
& =\underline{200} \text { so } 9 \% \text { of } 200 g=9 \times 2 g \\
& 100=18 g \\
& =2 g
\end{aligned}
$$

## Percentages: Non-Calculator

## Non- Calculator Methods

## Combining Methods

The previous 2 methods can be combined so allowing us calculate any percentage.
Example 4 Find $23 \%$ of $£ 15000$
Level 3


Example 5 Calculate the sale price of a computer which costs $£ 650$ and has a 15\% discount

$$
\begin{aligned}
10 \% \text { of } £ 650 & =£ 65 \\
5 \% \text { of } £ 650 & =£ 32.50 \\
\text { so } 15 \% \text { of } £ 650 & =£ 65+£ 32.50=£ 97.50 \\
\text { Total price } & =£ 650-£ 97.50=£ 552.50
\end{aligned}
$$

## Percentages: Calculator

## Calculator Method

To find the percentage of a quantity using a calculator, change the percentage to a decimal, then multiply.

Example 1 Find $23 \%$ of $£ 15000$
Level 3

$$
\begin{aligned}
& 23 \% \text { of } £ 15000 \\
= & \frac{23}{100} \times 15000 \\
= & 0.23 \times £ 15000 \\
= & £ 3450
\end{aligned}
$$



We NEVER use the \% button on calculators.
The methods taught in the mathematics department are all based on converting percentages to decimals.

Example 2 House prices increased by 19\% over a one year period.
Level $3 \quad$ What is the new value of a house which was valued at $£ 236000$ at the start of the year?

Increase $=19 \%$ of $£ 236000$
$=\frac{19}{100} \times 236000$
$=0.19 \times £ 236000$
$=£ 44840$

Value at end of year = original value + increase
$=£ 236000+£ 44840$
= £280 840

The new value of the house is $£ 280840$

## Percentages: One Quantity as a \% of Another

Finding the percentage


## a $\times 100 \Rightarrow \%$ <br> b

Example 1 There are 30 pupils in Class 3A3. 18 are girls. What percentage of Level $3 \quad$ class $3 A 3$ are girls?

18 out of 30 are girls

$$
\begin{aligned}
& \frac{18}{30} \times 100 \\
= & 0.6 \times 100 \\
= & 60 \% \quad \text { of } 3 \mathrm{~A} 3 \text { are girls }
\end{aligned}
$$

Example 2 James scored 36 out of 44 his biology test. What is his Level 3 percentage mark?

$$
\begin{aligned}
\text { Score } & =\frac{36}{44} \times 100 \\
& =0.81818 \ldots \times 100 \\
& =81.818 . . \% \\
& =82 \% \quad \text { (see rounding) }
\end{aligned}
$$

Example 3 In class 1×1, 14 pupils had brown hair, 6 pupils had blonde hair, 3 Level 3 had black hair and 2 had red hair. What percentage of the pupils were blonde?

Total number of pupils $=14+6+3+2=25$
6 out of 25 were blonde.

$$
\begin{aligned}
& \frac{6}{25} \times 100 \\
= & 0.24 \times 100 \\
= & 24 \% \quad \text { were blonde }
\end{aligned}
$$



Example 2 To make a fruit drink, 4 parts water is mixed with 1 part of cordial. Level 3

The ratio of water to cordial is $4: 1$
The ratio of cordial to water is $1: 4$

Example 3 In a bag of balloons, there are 5 pink, 7 blue and 8 yellow balloons. Level 3

The ratio of pink: blue : yellow is $5: 7: 8$

## Simplifying Ratios



Example 4 Purple paint can be made by mixing 10 tins of blue paint with 6 tins of Level 3 red. The ratio of blue to red can be written as $10: 6$

$\mathbf{B} B \mathbf{B} \mid \mathbf{B}: \mathbf{R} \mathbf{R}$

## Simplifying Ratios

Example 5 Simplify each ratio:
Level 3
Divide all by 3 .
(c) $6: 3: 12$
(a)
$4: 6$
(b)
$24: 36$

$\div 12\binom{24: 36}{2: 3} \div 12$
6: 3:12
2: 1:4

Example 6 A ruler costs $£ 1.20$ and a pencil costs 40 p.
Level 3
What is the ratio of the cost of a pencil to the cost of a ruler?

> pencil : ruler


Example 7 On a map 1 cm represents 500 m . Write this as a ratio.
Level 3


Ratio 1:50000


## Using ratios

Example 8 The ratio of fruit to nuts in a chocolate bar is $3: 2$.
Level 3 If a bar contains 15 g of fruit, what weight of nuts will it contain?


Whatever you do to one side you do the same to the other side. ( $\times 5$ )

## Sharing in a given ratio

Example Lauren and Sean earn money by washing cars. By the end of the day they have made £90. As Lauren did more of the work, they decide to share the profits in the ratio 3:2.
How much money did each receive?
Step 1
Total number of parts $=3+2$ $=5$

Using the ratio $3: 2$ add up the numbers to find the total number of parts.

Step 2

$$
\begin{aligned}
1 \text { part } & =90 \div 5 \\
& =£ 18
\end{aligned}
$$

Divide the total by the total number of parts (step 1) to find the value of 1 part.


Step 4

$$
£ 54+£ 36=£ 90
$$

 CHECK: add the answers to get back to the total.

Lauren received $£ 54$ and Sean received $£ 36$

Two quantities are said to be in direct proportion if when one doubles the other doubles etc.
We can use proportion to solve problems.

It is often useful to make a table when solving problems involving proportion.

Example 1 A car factory produces 1500 cars in 30 days. How many cars would they Level 3 produce in 90 days?


The factory would produce 4500 cars in 90 days.

Example 2 The Davidson's are off to France.
Level 3
The exchange rate is 1.4 euros for a $£ 1$. How many euros do they get for £500?


They get 700 euros for $£ 500$

Example 35 apples cost £2.25. How much do 8 apples cost?
Level 3


8 apples cost $£ 3.60$

It is sometimes useful to display information in graphs, charts or tables.

Example 1 The table below shows the average maximum temperatures (in degrees Level 2 Celsius) in Barcelona and Edinburgh.

|  | $J$ | $F$ | $M$ | A | M | J | J | A | S | O | N | D |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Barcelona | 13 | 14 | 15 | 17 | 20 | 24 | 27 | 27 | 25 | 21 | 16 | 14 |
| Edinburgh | 6 | 6 | 8 | 11 | 14 | 17 | 18 | 18 | 16 | 13 | 8 | 6 |

The average temperature in June in Barcelona is $24^{\circ} \mathrm{C}$


Frequency tables are used to collect and present data.
Often, but not always, the data is grouped into intervals.

Example 2 Homework marks for Class 4B
Level 2

$$
\begin{array}{lllllllllllllll}
27 & 30 & 23 & 24 & 22 & 35 & 24 & 33 & 38 & 43 & 18 & 29 & 28 & 28 & 27 \\
33 & 36 & 30 & 43 & 50 & 30 & 25 & 26 & 37 & 35 & 20 & 22 & 24 & 31 & 48
\end{array}
$$

| Mark | Tally | Frequency |
| :--- | :--- | :---: |
| $16-20$ | $H$ | 2 |
| $21-25$ | HI\\| \|I | 7 |
| $26-30$ | HI\\| \|\| | 9 |
| $31-35$ | $\\|\\|\\|$ | 5 |
| $36-40$ | $\\|\\|$ | 3 |
| $41-45$ | $\\|$ | 2 |
| $46-50$ | $\\|$ | 2 |

Each mark is recorded in the table by a tally mark.
Tally marks are grouped in 5's to make them easier to read and count.

Information Handling : Bar Graphs

Bar graphs are often used to display data. The horizontal axis should show the categories or class intervals, and the vertical axis the frequency. All graphs should have a title, and each axis must be labelled.

Example 1 How do pupils travel to school?
Level 2


When the horizontal axis shows categories, rather than grouped intervals, it is common practice to leave gaps, of equal size, between the bars. All bars should be of equal width. Numbers on the vertical axes should go up evenly.

Example 2 The graph below shows the homework marks for Class 4B.
Level 2


All bars should be of equal width.
Numbers on the vertical axes should go up evenly.

Line graphs consist of a series of points which are plotted, then joined by a line. All graphs should have a title, and each axis must be labelled. The trend of a graph is a general description of it.

Example 1 The graph below shows Heather's weight over 14 weeks as she follows an exercise programme.


The graph shows a decreasing trend.
Her weight has decreasing over the course of the 14 weeks.
Numbers on the both axes should be spaced evenly.

Example 2 Graph of temperatures in Edinburgh and Barcelona.
Level 2

Temperature in ${ }^{\circ} \mathrm{C}$


Numbers and/or categories on the axes should be spaced evenly.

A scatter diagram is used to display the relationship between two variables.
A pattern may appear on the graph. This is called a correlation.

Example 1 The table below shows the height and arm span of a group of first year

Level 4 and beyond boys. This is then plotted as a series of points on the graph below.

| Arm <br> Span <br> (cm) | 150 | 157 | 155 | 142 | 153 | 143 | 140 | 145 | 144 | 150 | 148 | 160 | 150 | 156 | 136 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Height <br> $(\mathrm{cm})$ | 153 | 155 | 157 | 145 | 152 | 141 | 138 | 145 | 148 | 151 | 145 | 165 | 152 | 154 | 137 |

The graph shows a general positive (slopes up from left to right) trend. As the arm span increases, the height also increases. This graph shows a positive correlation between arm span and height.


The line of best fit can be used to provide estimates.
For example, a boy of arm span 150 cm would be expected to have a height of around 151 cm .


A pie chart can be used to display information. Each sector (slice) of the chart represents a different category. The size of each category can be worked out as a fraction of the total using the number of divisions or by measuring angles.

Example 130 pupils were asked the colour of their eyes. The results are shown in the pie chart below.

Eye colour of 30 S1 pupils


How many pupils had brown eyes?

The pie chart is divided up into ten parts, so pupils with brown eyes represent $\frac{2}{10}$ of the total.
$\frac{2}{10}$ of 30
$=30 \times 2$
10
$=6$ so 6 pupils had brown eyes.


If no divisions are marked, we can work out the fraction by measuring the angle of each sector.

Level $3 \quad$ The angle in the brown sector is $72^{\circ}$.
so the fraction of pupils with brown eyes is $\frac{72}{360}$
$\frac{72}{360}$ of 30.
$=\frac{72}{360} \times 30$
$=6$ pupils
If you find a number of pupils for each eye colour using the same method as above the total should be 30 pupils.

## Information Handling : Pie Charts

## Drawing Pie Charts



On a pie chart, the size of the angle for each sector is calculated as a fraction of $360^{\circ}$.

Example 2 In a survey about television programmes, a group of people were asked Level 3 what was their favourite soap. Their answers are given in the table below. Draw a pie chart to illustrate the information.

| Soap | Number of people |
| :--- | :---: |
| Eastenders | 28 |
| Coronation Street | 24 |
| Emmerdale | 10 |
| Hollyoaks | 12 |
| None | 6 |
| Total number |  |

Total number of people $=80$

| Eastenders | $=\frac{28}{80} \times 360^{\circ}=126^{\circ}$ |  |
| :--- | :--- | :--- |
| Coronation Street | $=\frac{24}{80} \times 360^{\circ}=108^{\circ}$ |  |
| Emmerdale | $=\frac{10}{80} \times 360^{\circ}=45^{\circ}$ | $=\frac{12}{80} \times 360^{\circ}=54^{\circ}$ |
| Hollyoaks | $=\frac{6}{80} \times 360^{\circ}=\frac{27^{\circ}+}{360^{\circ}}$ | Check that the total <br> is $360^{\circ}$ by adding up <br> all the answers |
| None |  |  |




Information Handling : Averages


To provide information about a set of data, the average value may be given. There are 3 ways of finding the average value - the MEAN, the MEDIAN and the MODE.

## Mean

The mean is found by adding all the data together and dividing by the number of values.

## Median

The median is the MIDDLE value when all the data is written in numerical order (if we have middle pair of values, the median is half-way between these values).

## Mode

The mode is the value that occurs MOST often.

## Range

The range of a set of data is a measure of spread.


Range $=$ Highest value - Lowest value
Example 1 Class 1R2 scored the following marks for their homework assignment.
Level 2 Find the mean, median, mode and range of the results.
7. 9, 7,
5, 6, 7,
10,
9.
8,
, 8, 5,
8, 10

MEAN
Mean $=7+9+6+5+6+7+10+9+8+4+8+5+8+10$
14
$=\frac{102}{14}$
= 7.28571 ...
$=7.3$ (1.d.p.)
Level 3

MEDIAN - middle

$$
\begin{aligned}
& \begin{aligned}
& \text { Ordered values: } 4,5,5,6,6,7,7,8,8,8,9,9,10,10 \\
& \text { Median }=\frac{7+8}{2} \\
&=\frac{15}{2}=7.5 \\
& \begin{array}{l}
\text { This is a middle pair. } \\
\text { A single value would } B E \text { the median. } \\
\text { No calculation necessary. }
\end{array}
\end{aligned}
\end{aligned}
$$

MODE - most popular
8 is the most frequent mark, so Mode $=8$

Range
Range $=10-4=6$

| Impossible | Evens <br> $50 / 50$ chance |  | Certain |  |
| :---: | :---: | :---: | :---: | ---: |
|  | unlikely | 0.5 | likely | 1 |

Level 2


Example 1 What is the probability of rolling a 4?
Level 3

$$
P(4)=\underline{1}
$$

$$
6
$$



Example 2 What is the probability of rolling an even number?
Level 3
$P($ even number $)=\frac{3}{6}=\frac{1}{2}$
Example 3 What is the probability of rolling number greater than 2?
Level 3
$(3,4,5,6)$

$$
P(>2)=\frac{4}{6}=\frac{2}{3}
$$



Probabilities can be expressed as a FRACTION or a DECIMAL and even if we want as a percentage.

Example 4 What is the probability of a tail when you toss a coin?
Level 3

$$
P(\text { tail })=\frac{1}{2}=0.5
$$



When making choices we need to consider:
the element of risk,
the probability of the event happening and the consequences of the event happening.

Probability


## Expectation $=P($ event $) \times$ Number of trials

Example 5 If I were to roll a die 300 times. How many 5's should I expect to get? Level 4

$$
P(5)=\frac{1}{6}
$$

Expected number of 5 's $=\frac{1}{6} \times 300$

$=\underline{300}$
6
$=50$

I should expect to roll a 5 fifty times.

| Term | Definition |
| :---: | :---: |
| Add; Addition (+) | To combine 2 or more numbers to get one number (called the sum or the total) <br> Example: $12+76=88$ |
| a.m. | (ante meridiem) Any time in the morning (between midnight and 12 noon). |
| Approximate | An estimated answer, often obtained by rounding to nearest 10, 100 or decimal place. |
| Axis |  |
| Calculate | Find the answer to a problem. It doesn't mean that you must use a calculator! |
| Data | A collection of information (may include facts, numbers or measurements). |
| Denominator | The bottom number in a fraction (the number of parts into which the whole is split). |
| Difference (-) | The answer to a subtraction calculation (amount between 2 numbers). Example: The difference between 50 and 36 is 14 $50-36=14$ |
| Digit | A single number. The digits are $0,1,2,3,4,5,6,7,8,9$ |
| Discount | Amount of money you save on an item. |
| Division ( $\div$ ) | Sharing a number into equal parts. $24 \div 6=4$ |
| Double | Multiply by 2. |
| Equals (=) | Makes or has the same amount as. |
| Equivalent fractions | Fractions which have the same value. <br> Example $\frac{6}{12}$ and $\frac{1}{2}$ are equivalent fractions |
| Estimate | To make an approximate or rough answer, often by rounding. |
| Evaluate | To work out the answer. |
| Even | A number that is divisible by 2. Even numbers end with $0,2,4,6$ or 8 . |
| Factor | A number which divides exactly into another number, leaving no remainder. <br> Example: The factors of 15 are $1,3,5,15$. |
| Frequency | How often something happens. In a set of data, the number of times a number or category occurs. |
| Greater than (>) | Is bigger or more than. Example: 10 is greater than 6. $10>6$ |
| Gross Pay | The amount of money you earn before any deductions are taken. |
| Histogram | A bar chart for continuous numerical values. |


| Increase | An amount added on. |
| :---: | :---: |
| Least | The lowest number in a group (minimum). |
| Less than (<) | Is smaller or lower than. Example: 15 is less than 21. $15<21$. |
| Maximum | The largest or highest number in a group. |
| Mean | The arithmetic average of a set of numbers (see p32) |
| Median | Another type of average - the middle number of an ordered set of data (see p32) |
| Minimum | The smallest or lowest number in a group. |
| Minus (-) | To subtract. |
| Mode | Another type of average - the most frequent number or category (see p32) |
| Most | The largest or highest number in a group (maximum). |
| Multiple | A number which can be divided by a particular number, leaving no remainder. <br> Example Some of the multiples of 4 are $8,16,48,72$ |
| Multiply (x) | To combine an amount a particular number of times. Example $6 \times 4=24$ |
| Negative Number | A number less than zero. Shown by a minus sign. Example -5 is a negative number. |
| Numerator | The top number in a fraction. |
| Odd Number | A number which is not divisible by 2 . Odd numbers end in $1,3,5,7$ or 9 . |
| Operations | The four basic operations are addition, subtraction, multiplication and division. |
| Order of operations | The order in which operations should be done. BODMAS (see p9) |
| Per annum | Per year. |
| Place value | The value of a digit dependent on its place in the number. Example: in the number 1573.4, the 5 has a place value of 100 . |
| p.m. | (post meridiem) Any time in the afternoon or evening (between 12 noon and midnight). |
| Prime Number | A number that has exactly 2 factors (can only be divided by itself and 1). Note that 1 is not a prime number as it only has 1 factor. |
| Product | The answer when two numbers are multiplied together. Example: The product of 5 and 4 is 20. |
| Quotient | The answer to a divide calculation. Usually we also have a remainder |
| Remainder | The amount left over when dividing a number. |
| Share | To divide into equal groups. |
| Sum | The answer to an add calculation (Total of a group of numbers). |
| Total | The sum of a group of numbers (found by adding). |

